PHYS 301
Electricity and Magnetism

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Today!

- Quiz!!
- Vector review
- The Gradient

Vectors: scalar (dot) product

$$\vec{A}\cdot\vec{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$$

or

$$\vec{A}\cdot\vec{B} = \sum_{i,j=1}^{3} A_i B_j \underbrace{\hat{e}_i \cdot \hat{e}_j}_{\delta_{ij}} = \sum_{i=1}^{3} A_i B_i$$

δ_{ij} = Kronecker delta

commutative and distributive, but not associative!

Why not?

Also $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

Vectors: vector (cross) product

$$\vec{A}\times\vec{B} = \hat{i}(A_y B_z - A_z B_y) + \hat{j}(A_z B_x - A_x B_z) + \hat{k}(A_x B_y - A_y B_x)$$

or

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

determinant

distributive, but not commutative

$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

nor associative

$\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$

Also $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{n}$

direction determined by RH Rule

Cross Product: Another form?

- For the scalar product, $\vec{A}\cdot\vec{B} = \sum_{i=1}^{3} A_i B_i$ is so cute and compact!
- For vector product, we can write that if

$$\vec{C} = \vec{A}\times\vec{B} \quad \text{then} \quad C_i = \sum_{j,k=1}^{3} \varepsilon_{ijk} A_j B_k$$

where the Levi-Civita symbol (pseudo-tensor) is

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{if i, j, k form an even permutation of 1,2,3} \\ 0 & \text{if any index = any other index} \\ -1 & \text{if i, j, k form an odd permutation of 1,2,3} \end{cases}$$

Levi-Civita (permutation) symbol

- In other words …

$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = +1$

$\varepsilon_{113} = \varepsilon_{232} = \varepsilon_{221} = 0$

$\varepsilon_{132} = \varepsilon_{213} = \varepsilon_{321} = -1$

- A useful property of the Levi-Civita symbol:

$$\sum_{k=1}^{3} \varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

Vectors: triple vector products

- There are several (see text)
- Most likely to use:

volume of parallelepiped

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

Position & Displacement Vectors

- Coulomb's Law: $\vec{F} = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{q_i Q}{\mathscr{r}_i^{\,2}}\hat{\mathscr{r}}_i$
- Position vectors: $\vec{r}_i$
- Displacement vectors: $\vec{\mathscr{r}}_i = \vec{r} - \vec{r}_i'$